Applicability of the Godunov's Method for Fundamental Four-Equation FSI Model

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ABSTRACT

The present paper addresses mathematical and numerical model needed for description of the axial pipe movement induced with transient fluid motion. This phenomenon is also known as Fluid-Structure Interaction (FSI). Standard Skalak's four-equation model was applied and solved with improved second-order accurate numerical method that is based on Godounov's upwind first-order accurate method. Special attention was made to applicability of the numerical method for solution of the mathematical model. The method was verified using standard Delft Hydraulics Benchmark Problem A, and the preliminary results are very promising.

1 INTRODUCTION

Conventional computer simulation of the fast transient in the pipeline network filled with liquid is usually conducted on the base of the assumption that the pipeline is absolutely stiff and fully supported (see comparisons of pressure history in stiff and extremely soft pipeline in Fig. (1)). Economical effectiveness as leading force and also safety requirements as regulatory force, demand higher accuracy than provided with conventional codes and methods. Although these codes are very efficient and verified, they lack appropriate models of FSI mechanisms. It is therefore necessary to develop efficient and easy-to-use mathematical and numerical models for simultaneous analyses of interactions between the fluid and structure. Since 1970's a substantial amount of research have been made in this field, focused on understanding and quantifying interaction between transient flow in the pipe and the resulting vibration of the pipeline structure. This work is summarized, analyzed and discussed in a really comprehensive reviews made by Tijseling and Wiggert in [1] and [2]. On the other hand, it is always possible to build stiff and fully anchored pipeline not sensitive to traveling pressure waves. Such structures would fulfill conventional assumption but they would have high potential to fail due to the temperature loads.

What is fast transient and what is the relation with FSI? Steady state flow is present under normal operating conditions in NPP or other industrial pipelines. Due to the inappropriate valve operation, at stop or start-up of the pumps, during the system temperature changes, during the pressure changes in the fluid, especially accidental like LOCA, fast transient flow is induced in the pipeline. There are many disturbances that are traveling along the pipe (pressure, velocity, temperature, density, etc.) with characteristic velocity that is approximately equal to the speed of sound. In the field of FSI, pressure waves are the most
important, since they significantly influence the flow with cavitation and condensation, and can induce loads and deformations of the structure. Sudden velocity changes have the same potential but they are correlated to pressure changes as describes Joukowsky equation [3]:

\[
\Delta p = \rho_f c_f \Delta v
\]

where \( p \) stands for pressure, \( v \) for velocity, \( \rho_f \) for fluid density and \( c_f \) for speed of sound.

Generally, any moving fluid induces pressure load on the structure. Burdened structure is deformed and the deformations are then actually obstacles for the moving fluid that causes redistribution of the pressure load. This dynamic phenomenon is the base of the FSI mechanism. Beside noise and significant displacements, extreme pressures and immediate failure or fatigue of the structure due to the vibration are possible consequences. The final stage of this phenomenon is pipe or support failure. Statistical data of the USA Office of Pipeline Safety for years 1986-2000 under column "Failed Pipe (Internal Force)" shows there have been a total of 5979 accidents, with 357 deaths and 3494 injuries, costing over $1 billion [4]. With appropriate FSI analysis followed by appropriate design and definition of operating procedures it is possible to reduce maximum pressure in the fluid and maximum stresses in the structure. It is further possible to change frequency of the system, it is possible to control energy transfer between the fluid and structure and finally, it is possible to prevent breakdown of the pipe or other, less severe failures.

![Figure (1) Pressure history comparison near the valve, for details see Chap. 3](image)

In contrary to the conventional two-step (uncoupled) FSI analyses, fully two-way coupled analyses have been taken into account very rarely in the past because discrepancies with conventional two-step analysis were seldom anticipated. There were neither efficient mathematical models nor numerical methods for available computers and finally, FSI analyses have been so expense that they were performed only for the most important pipelines [4]. In 1989 Wylie pointed out that 98% of the pipelines are not subjected to the significant FSI [2]. He stressed that since there is no convenient criterion for simple FSI inspection, it is necessary to make analysis for all pipelines. Casadei also recommends FSI analysis always if the fluid is incompressible (single-phase liquid flow) or if the structure is deformable [5]. Lavooij and Tijsseling proposed the first reliable criterion for inspection of the FSI in 1991 but it was validated only for single elbow pipeline [6].

The long-term goal of the authors of the present paper is to incorporate structural dynamics equations into the recently developed one-dimensional two-phase thermal-hydraulics code WAHA [7] and thus provide very efficient, accurate and easy-to-use two-phase FSI tool. With this purpose the study of the Skalak's fundamental four-equations single-phase model was performed and described in the present paper with special emphasis on the
applicability of the Godunov's numerical method for numerical integration of the axial structural dynamic equations. Chapter 2 provides detailed description of the mathematical model and chapter 3 contains description of the experimental setup and different initial and boundary conditions. Description of the numerical method with discussion and figures on numerical features can be found in the chapter 4.

2 SKALAK'S MATHEMATICAL MODEL

In 1956 Skalak [8] defined a set of four linear first-order partial differential equations, which are able to describe interactions between the fast transient in the fluid and axial movement of the straight section of the pipe. The model consists of four equations, governing fluid pressure $p$, fluid velocity $v$, axial pipe stress $\sigma_z$ and axial pipe velocity $u_z$, but it disregards two-phase flow, friction and damping effects. The effectiveness of this model has been proved extensively by both theoretical and experimental studies in time and frequency domain, and it has been widely applied in engineering practices and academic researches [9]. This model became fundamental model in the field of the FSI in liquid-filled pipe systems and it has been improved with additional structural dynamic equations for simulations of more complex pipelines.

The fluid part of the considered model is one-dimensional model with two first order partial differential equations. It is energy conserving (no damping and friction mechanism), single-phase (no cavitation considered), valid for pressure waves with low frequency (long wavelength approximation), and convective terms are neglected. The momentum equation is:

$$\frac{\partial v}{\partial t} + \frac{1}{\rho_f} \frac{\partial p}{\partial z} = 0$$

(2)

where $t$ stands for time, $z$ for axial position and $\rho_f$ for fluid density that is constant during the calculation. The continuity equation is:

$$\frac{\partial v}{\partial z} + \left( \frac{1}{K} + \frac{2R}{Ee} \right) \frac{\partial p}{\partial t} - \frac{\nu}{E} \frac{\partial \sigma_z}{\partial t} = 0$$

(3)

where $R$ stands for internal radius of the pipe, $e$ for pipe thickness, $\nu$ for Poisson's ratio, $E$ for Young's elasticity modulus and $K$ for fluid bulk modulus.

The following two equations describe propagation of the axial stress waves in the straight, thin-walled, linearly elastic pipe of circular cross-section under the influence of pressure variation in the fluid. The one-dimensional two-equation first order partial differential equations are derived from axial wave equation for the pipe:

$$\frac{\partial u_z}{\partial t} - \frac{1}{\rho_i} \frac{\partial \sigma_z}{\partial z} = 0$$

(4)

$$\frac{\partial u_z}{\partial z} - \frac{1}{E} \frac{\partial \sigma_z}{\partial t} + \frac{\nu R}{Ee} \frac{\partial p}{\partial t} = 0$$

(5)

where the only unknown variable yet is $\rho_i$ that stands for pipe wall density.

Generally there are four types of waves that characterize FSI: axial, flexural and torsional stress waves in the pipeline and pressure waves in the fluid [2]. The proposed four-equation model (Eqs. (2) - (5)) is able to describe axial stress waves and pressure waves in the fluid. These waves are coupled with distributed Poisson coupling mechanism where pressure
waves in the fluid are coupled with axial waves in the structure via radial deformations and local *junction coupling* mechanism where axial stress waves are appropriately coupled with pressure waves at boundary conditions. Poisson coupling is figuratively known also as pipe breathing.

The system of equations can be written in the following vectorial form:

\[
\mathbf{A} \frac{\partial \mathbf{\Psi}}{\partial t} + \mathbf{B} \frac{\partial \mathbf{\Psi}}{\partial z} = 0
\]

where

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \left( \frac{1}{K} + \frac{2R}{E \varepsilon} \right) & 0 & -\frac{2\nu}{E} \\
0 & 0 & 1 & 0 \\
0 & \frac{\nu R}{E \varepsilon} & 0 & -\frac{1}{E}
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{\rho_t} \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
\mathbf{\Psi} = \begin{bmatrix}
\nu \\
p \\
\dot{u}_z \\
\sigma_z
\end{bmatrix}
\]

With few matrix operations, the system can be rearranged into the following linear hyperbolic system:

\[
\frac{\partial \mathbf{\Psi}}{\partial t} + \mathbf{A}^{-1} \cdot \mathbf{B} \frac{\partial \mathbf{\Psi}}{\partial z} = \mathbf{C} \frac{\partial \mathbf{\Psi}}{\partial t} + \mathbf{L} \cdot \mathbf{L}^T \frac{\partial \mathbf{\Psi}}{\partial z} = 0
\]

The Jacobian matrix \(\mathbf{C}\) has some very important properties. It is diagonalisable i.e., it is possible to analytically define eigenvalues \(\Lambda\) and eigenvectors \(\mathbf{L}\). And the most important although not the most correct, the eigensystem is constant during the simulation due to the assumption of the single-phase flow and constant fluid density. These assumptions are generally not accurate because traveling pressure waves generate local and distributed cavitation along the pipe and influence the fluid density. Thus, the speed of sound and therefore the eigenvalues and speed of traveling pressure waves alter in real systems. For the purpose of the planned test calculations and according to the reports of several practitioners [1], for simulations of single-phase transients in cold liquid, the contribution of the assumption to the overall error is acceptable.

### 3 THE EXPERIMENT

Several benchmark problems (Delft Hydraulics Benchmark (DHB) Problems A to F) have been defined to test numerical methods and FSI software [9]. Experimental data do not exist so these are only benchmark problems. Problem A that concerns simple Tank-pipe-valve system (Fig. (2)) has been chosen for calculations performed in this paper. According to the boundary conditions (valve is fixed or free, tank is always fixed) and applied coupling mechanism the Problem A is further divided into 5 cases. In all cases, initial steady state flow is present in the pipe in direction from the tank to the valve. At time \(t = 0\) s the valve is instantaneously closed and the transient flow is induced. In case 1, the valve is fixed (zero pipe displacement and velocity in axial direction) and the only available Poison coupling mechanism is taken into account. Cases 2 to 4 have unrestrained valve in axial direction and all combinations of available coupling mechanisms are used. Case 5 was made only for
comparisons; the results are equal to conventional calculation of fluid transient without FSI. Note that pressure changes are relative, thus negative pressures are possible.

Pipe properties:
- \( L = 20 \text{ m} \)
- \( R = 398.5 \text{ mm} \)
- \( e = 8 \text{ mm} \)
- \( E = 210 \text{ GPa} \)
- \( \nu = 0.3 \)
- \( \rho_s = 7900 \text{ kg/m}^3 \)

Initial conditions:
- \( v = 1 \text{ m/s} \)
- \( p = 0 \text{ Pa} \)
- \( \rho_f = 1000 \text{ kg/m}^3 \)

Boundaries conditions (instantaneous valve closure in all cases):
- CASE 1, Tank and valve are fixed (poisson coupling only)
- CASE 2, Tank is fixed, valve is free in axial direction (poisson coupling only)
- CASE 3, Tank is fixed, valve is free in axial direction (junction coupling only)
- CASE 4, Tank is fixed, valve is free in axial direction (poisson and junction coupling)
- CASE 5, No coupling (tank and valve are fixed, pipe is stiff)

Figure (2) Geometry, initial and boundary conditions

Figure (1) shows pressure history in the measuring point, located as close as possible to the valve. Pressures of two basic cases (1 and 4) are compared to the case 5 that correspond to the conventional analysis where no FSI is taken into account. DHB problem A has been defined in such manner that FSI influence is clear and evident, thus the authors suppose that such 'soft' pipelines are not used in practice. In case 1 (left diagram) the valve is fixed and the distance between the tank and valve is constant. Only Poisson coupling mechanism is possible in such case and the diagram shows that the waterhammer in soft pipe increase maximal pressure near the valve for more than 80%. The maximal pressure appears later on and is not evident from the present diagram. The authors estimate that damping and wall friction that are not taken into account in our calculations would prevail in real system and the maximal achieved pressure wouldn't exceed significantly conventionally defined pressure.

Different conclusion can be made from the right diagram (case 4, junction and Poisson coupling mechanisms) where one can see that maximum is achieved in the second pressure rise recorded near the valve. Damping and friction losses are here not important - this is one of the fundamental issues of the fast transients. Maximal pressure exceeds conventional pressure for over 50% and it cannot be neglected during the pipeline design process.

Figure (3) Pressure history and valve displacement (coupling mechanisms)

Figure (3) shows comparison of different coupling mechanisms - cases 2 to 4. This figure is interesting because it shows that interaction of both mechanisms (Poisson and
junction) is crucial. The conclusion is that both particular mechanisms must be included in simulation of the proposed pipeline to obtain correct maximal design pressure. Right diagram shows axial displacement of the valve. Although the particular lines are not comparable between each other, the maximal displacements are similar in all cases (case 5: valve is fixed).

4 GODUNOV'S NUMERICAL METHOD

In the linear theory for conservation laws it is assumed that the solutions are smooth. But, the solutions of our interest are discontinuous (as sharp as possible) i.e. we are solving multiple Riemann problem. Typical behaviour of the first-order numerical methods near discontinuous solutions is that they give very smeared solutions, while the second-order methods give oscillations. One of the common and very interesting one-sided (upwind) first-order, explicit and 2-level numerical methods is Godunov's method. Because one-sided methods cannot be used for systems of equations with eigenvalues ($\lambda_p$) of mixed sign, the Godunov's method has its base on appropriate splitting between wave propagation to the left (superscript -) and right (superscript +):

$$
\lambda_p^+ = |\lambda_p| \cdot f_p^+ \quad \text{where} \quad f_p^+ = \max\left(0, \frac{\lambda_p}{|\lambda_p|}\right) \quad \text{and} \quad \Lambda^+ = \text{diag}(\lambda_1^+, \ldots, \lambda_4^+)
$$

$$
\lambda_p^- = |\lambda_p| \cdot f_p^- \quad \text{where} \quad f_p^- = \min\left(0, \frac{\lambda_p}{|\lambda_p|}\right) \quad \text{and} \quad \Lambda^- = \text{diag}(\lambda_1^-, \ldots, \lambda_4^-)
$$

(9)

Index $p$ is running from 1 to 4. Taking into account $C^+ = L \cdot \Lambda^+ \cdot L$, $C^- = L \cdot \Lambda^- \cdot L$ and $C^+ + C^- = C$ the equation (8) can be also written as:

$$
\frac{\partial \psi}{\partial t} + C^+ \frac{\partial \psi}{\partial z} + C^- \frac{\partial \psi}{\partial z} = 0
$$

(10)

The Jacobian matrix $C$ shall be evaluated in a staggered grid, but since the eigensystem is constant along the pipe during the simulation, averaging and indexes $C_{j-1/2}$ and $C_{j+1/2}$ are omitted here for simplicity.

As mentioned before, problems of the one-sided first-order accurate discretisation are smeared solutions in the vicinity of the discontinuities. LeVeque proposed solution of the problem, based on Godunov's method with a combination of the first- and the second-order accurate discretisation [10]. A part of the second-order discretisation is determined by the limiters $\phi_p$, which "measure" the smoothness of the solutions. If the solutions are smooth, larger part of the second-order discretisation is used, otherwise larger part of the first-order discretisation is used. Elements ($\lambda_p$) of the diagonal matrices $\Lambda^+$, $\Lambda^-$ in Equations (9) are thus evaluated using modified factors $f_p^+$ and $f_p^-$ as follows:

$$
\lambda_p^+ = |\lambda_p| \cdot f_p^+ \quad \text{where} \quad f_p^+ = \max\left(0, \frac{\lambda_p}{|\lambda_p|}\right) + \frac{\phi_p}{2} \left(\frac{|\Delta \lambda_p|}{\Delta x} - 1\right)
$$

$$
\lambda_p^- = |\lambda_p| \cdot f_p^- \quad \text{where} \quad f_p^- = \min\left(0, \frac{\lambda_p}{|\lambda_p|}\right) - \frac{\phi_p}{2} \left(\frac{|\Delta \lambda_p|}{\Delta x} - 1\right)
$$

(11)

The first term of the factors $f_p^+$ and $f_p^-$ is the first-order upwind discretisation, and the second term with limiters $\phi_p$ is the second-order correction. The flux limiter $\phi_p$ is calculated using one of the following expressions [10]:

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MINMOD: \( \phi_p = \max(0, \min(1, \theta_p, j)) \)  
Van Leer: \( \phi_p = \frac{\left| \theta_p \right| + \theta_p}{1 + \left| \theta_p \right|} \)  
Superbee: \( \phi_p = \max(0, \min(2 \theta_p, 1), \min(\theta_p, 2)) \)

where \( \theta_p \) measures the ratio of the left and the right gradients of the characteristic variables \( \delta \bar{\xi} = \mathbf{L}^{-1} \delta \bar{\psi} \) that are needed only for evaluation of \( \theta_p \). Characteristic variables correspond to characteristic-like form of Eq. (10):

\[
\frac{\partial \bar{\xi}}{\partial t} + \Lambda^+ \frac{\partial \bar{\xi}}{\partial z} + \Lambda^- \frac{\partial \bar{\xi}}{\partial z} = 0
\]

\( \theta_p \) in the midpoint \( j + 1/2 \) is thus evaluated as:

\[
\theta_{p,j+1/2} = \frac{\bar{\xi}_{p,j+1,m} - \bar{\xi}_{p,j-m}}{\bar{\xi}_{p,j+1} - \bar{\xi}_{p,j}}, \quad m = \frac{\lambda_p}{|\lambda_p|}, \quad p = 1, 4
\]

Figure (4)  Case 1: Pressure history near the valve - influence of the limiters \((N = 1000)\)

Figure (5)  Case 4: Pressure history near the valve - influence of the limiters \((N = 100)\)

Figures (4) and (5) show that the steepest waves are obtained with the Superbee limiter, while the most smeared waves (but still second-order accurate) are obtained with the MINMOD limiter. Solutions obtained with the Van Leer limiter lie between the solutions obtained with the
MINMOD and the Superbee limiters. The first-order accurate characteristic upwind scheme is obtained if the values of the limiters $\phi_p$ are set to zero.

Finally, the applied difference scheme is as follows:

$$\psi_j^{n+1} = \psi_j^n - C^* \left( \psi_j^n - \psi_{j-1}^n \right) \frac{\Delta t}{\Delta x} - C \left( \psi_{j+1}^n - \psi_j^n \right) \frac{\Delta t}{\Delta x} = 0 \quad (17)$$

Each numerical method is a source of certain error known as numerical damping. Figure (6) shows three cases where our code was used for long-term simulation of the transient. The same initial and boundary conditions (Case 4) can yield absolutely different results due to the slight differences in numerical approaches (limiters). First-order numerical method emulates physical behaviour. There is no mathematical model of damping or friction losses integrated into the code but the simulation shows that transient is damped. In contrary, results obtained with Van Leer (and Superbee) limiters are amplified over the reasonable limits. Finally, simulation with MINMOD limiter shows very modest damping that is very close to anticipated transient course according to the assumptions of the mathematical model.

![Figure (6) Case 4: Pressure history near the valve - numerical damping](image)

As figures (4) and (5) indicates, grid refinement study is always an important issue while discussing effectiveness of the numerical method. One of the conditions for use of the proposed method is Courant-Friedrichs-Levy condition for stability defined as:

$$\Delta t < \frac{\min \Delta x}{\max \left( \lambda_p \right)} \quad (18)$$

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Consequences of this condition on calculation are two-fold. First, if number of the computational volumes \( N \) is increased therefore calculation time-step is decreased. This yields in practice significant increase in time consumption of the processor. For instance: calculation of the first 0.2 seconds of the case 4 transient on P4 3.0 GHz processor on coarse grid with \( N = 100 \) volumes only 2 seconds are needed, while for the problem with grid with \( N = 1000 \) volumes one needs 120 seconds. The second consequence affects accuracy of the results as is evident from Figs. (4), (5) and (7). Our conclusion is that the number of computational volumes is very important and for presentation of the final results it is necessary to use the (reasonable) finest grid while for temporary results and comparisons, coarsest grids are sufficient. Figures (4) and (5) show also that grid refinement affects also influence of the different limiters. It is not important which limiter to use on dense grid, while on coarse grid results obtained with Superbee limiters are most reliable.

![Figure (7) Case 4: Pressure history near the valve - grid refinement study (Superbee limiter)](image)

5 CONCLUSIONS

The present paper has pointed out the importance of the FSI in pipelines and the need to provide efficient and easy-to-use FSI analysis software. With this purpose the study of the basic Skalak's four-equation mathematical model was presented and solved with second-order accurate numerical method based on Godunov's method. The results show that the proposed numerical method is very efficient, robust and accurate although it has some deficiencies. These deficiencies are easy to by-pass if the user is aware of them. It is necessary to stress again that this numerical approach is intended for calculation of fast transients where only the very first moments of the transient are in focus where numerical damping is still negligible. Authors are aware that this is just theoretical study, where results are comparable to the results of similar codes, but the problem is theoretical. Full verification of the code will be possible only with simulations and comparisons to the measurements and this is the goal for the future work.

REFERENCES


